How to Find the Church Festivals.

BARNARD.

HOW TO FIND

THE CHURCH FESTIVALS

WITHOUT TABLES;

BEING A LETTER ADDRESSED TO THE CHAIRMAN OF THE COMMITTEE ON THE PRAYER BOOK IN THE HOUSE OF CLERICAL AND LAY DEPUTIES OF THE GENERAL CONVENTION OF THE CHURCH, IN 1871.

BY

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LETTER.

COLUMBIA COLLEGE, NEW YORK,
April 5, 1871.

REVEREND AND DEAR SIR,

At your request I here repeat the substance of a communication which I ventured to address to you during the session of the General Convention of the Church in 1868.

In the introduction to the Prayer Book there are given tables exhibiting the time of Easter for one or two cycles of the moon; and also rules for computing the time when Easter will occur in years beyond the limits of the tables. These tables are very well, and serve the purposes of most persons. But for one who desires to know on what day Easter occurred in some year long past, or on what day Easter will occur in some future year not embraced in the tables of the Prayer Book, the rules of computation, as given, are not so simple as could be desired, nor are they easy of application without the use of pen or pencil. The object of my communication of 1868 was to suggest the inquiry whether, in the new edition of the Prayer Book, it would not be advisable to introduce some simple rules, easily fixed in the memory, by means of which the place of Easter in the calendar of any year, past or future, may be found mentally, and that without any very painful effort. Such rules I have been accustomed myself, for a long time, to use; and I furnish them herewith for your examination.

The place of Easter depends, first, upon the place of the Paschal full moon, and secondly, upon the Dominical or Sunday Letter.

The place of the Paschal full moon depends upon the Golden Number, or the place of the year in the lunar cycle; and upon another term which, throughout an entire century, and sometimes throughout two or three entire centuries, is constantly the same, but which sometimes also changes by a unit in passing from century to century. This change may be called the Secular Correction.

In order to find Easter we must, therefore, be able to find, first, the Golden Number; secondly, the Sunday Letter; and thirdly, the *constant* term, as modified by the Secular Correction. This latter is unimportant if the Easter sought falls within our century, since the *constant* for this century may be learned once for all.

It may be observed, in general, in regard to the rules which follow, that the number denoting any given year of our Lord may be resolved into two parts, one of them being a certain number of complete centuries, and the other a certain number of years of an incomplete century. These two parts are treated separately, and the results combined. Thus, 1871 is divided into 18 and 71.

1. TO FIND THE GOLDEN NUMBER.

Divide the years by TWENTY, *and add the* QUOTIENT *to the remainder.*

Divide the centuries by* FOUR, *and add the* QUOTIENT *to* FIVE TIMES *the* REMAINDER.

Add together the two results, and increase the sum by ONE. *If the final result is* NINETEEN, *or less, it is the Golden Number. If not, subtract* NINETEEN, *and the remainder is the Golden Number.*

EXAMPLE.—What is the Golden Number in 1871? $18 \div 4 = 4$, with 2 remainder. And $4 + 5 \times 2 = 14$; $71 \div 20 = 3$, with 11 remainder. And $3 + 11 = 14$. Finally, $14 + 14 + 1 = 29$. And $29 - 19 = 10$, Golden Number.

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* If the centuries exceed *nineteen*, drop nineteen as often as possible before proceeding with this division.

2. To Find the Sunday Letter.

According to the rules of the Gregorian calendar, the centurial years which are exact multiples of 400 are leap years, and the three intermediate centurial years have only three hundred and sixty-five days each. In every succession of four centurial years, beginning with the leap year centurial, the Dominical Letters return in the following order, viz., A, C, E, G. Or giving (as is most convenient) to the letters the numerical values which correspond to their places in the alphabet, they present the series, 1, 3, 5, 7. This simple series is easily remembered. The number in this series which corresponds to the *hundreds* of a given year may then be called the *centurial* belonging to that date.

If the years of the incomplete century be divided into *twenties*, and the excess of twenties resolved into *fours*, the Dominical Letter will advance *three* places for every twenty, *two* places for every four, and *six* places for every unit of the still outstanding remainder. Hence the Sunday Letter will be found by taking the sum of four numbers, which may be called the *centurial*, the *vigesimal*, the *quaternial*, and the *residual*. And hence the rule:

In the series 1, 3, 5, 7, *find the number corresponding to the given century* (*remembering that the number* 1 *belongs to the century divisible by* FOUR) *and call this the* CENTURIAL.

Multiply the twenties of the incomplete century by THREE, *and call the product the* VIGESIMAL.

Multiply the fours in the excess of twenties by TWO, *and call the product the* QUATERNIAL.

Multiply the final remainder by SIX, *and call the product the* RESIDUAL.

If the sum of the numbers thus obtained is SEVEN, *or less, it is the numerical value of the Sunday Letter. If it be greater than seven, subtract seven as often as may be necessary to reduce it to seven or below, and the final result is the Sunday Letter.**

EXAMPLE.—What is the Sunday Letter of 1871 ?

1st. For the *centurial.* The first century of the quadricentennium being 16, the third is 18, of which the *centurial* is 5.

2d. There are *three* twenties in 71, and $3 \times 3 = 9 =$ the *vigesimal.*

3d. There are *two* fours in 11 (71–60), and $2 \times 2 = 4 =$ the *quaternial.*

4th. The final remainder (or excess of fours in 11) is 3. and $3 \times 6 = 18$ $=$ the *residual.*

Then $5 + 9 + 4 + 18 = 36$, and $36 - 5 \times 7 = 1 = A$.

NOTE.—That in summing up the terms, the *sevens* may be dropped during the operation, thus simplifying the solution: $9 + 5 = 14$, for instance, being equal to twice seven, may be disregarded; and from 18, fourteen may be dropped. The final summation will then be $4 + 4 = 8$, and $8 - 7 = 1 =$ A.

The foregoing rule may be kept in mind by means of the following formula, in which Δ stands for the value of the Dominical Letter. C, V, Q, and R are the initials of the terms employed above, and n denotes any number:

$$\Delta = C + 3\,V + 2\,Q + 6\,R - 7\,n.$$

ALTERNATIVE RULE.—The following process, though involving occasionally larger numbers, may be found by some persons more easy of application. Let it first be understood that the difference between *seven* and any number less than seven, is called the *complement* of that number.

Then, the *centurial* is found as before; the *vigesimal* is discarded, and the *quaternial* is one half the number expressing the latest leap year in the incomplete century. Thus, for 1797, the centurial is three, and the quaternial is one half of 96 (the latest leap year in 97); that is, it is 48. The *resid-*

* To find the Sunday Letter in old style, the process is the same, except in respect to the *centurial.* The centurial for old style is found by adding *three* to the number of the complete centuries, and suppressing *seven* from the result as often as possible. The vigesimal, quaternial, and residual are the same for both old style and new.

ual, as under the former rule, is *one*, and its complement is *six*. Add then the centurial, the quaternial, and the complement of the residual, viz., 3 + 48 + 6 = 57; and suppress the *sevens*, viz., 7 × 8 = 56; and there remains 1 = A = the Sunday Letter of 1797. Hence the concise rule:

To the centurial add one half the number of the largest leap year in the incomplete century, and the complement of the years in excess of leap year. The sum suppressing sevens, is the value of the Dominical Letter.

3. To Find the Value of the Constant* used in determining Paschal Full Moon, as modified by the Secular Correction.

From the number of the centuries take its fourth part and its third part (disregarding fractions in both cases), and increase the result by TWO.

Example.—What is the constant for 1871?

$$18 \div 4 = 4. \quad 18 \div 3 = 6. \quad \text{And } 18 - 4 - 6 + 2 = 10.$$

The constant is the same for every year of the century to the 99th, but exclusive of the 100th.

What is the constant for 2371?

$$23 \div 4 = 5. \quad 23 \div 3 = 7. \quad \text{And } 23 - 5 - 7 + 2 = 13.\dagger$$

This rule is true up to the year 4200; but in that year, and those following, the number of the century must be diminished by *one* before taking the third part. In other respects the rule remains unaltered. And in the year 6700, and subsequently, up to 9200, the number of the century must be diminished by *two* before taking the third part. In the year 9200, and subsequently, up to the year 11,700, the rule will be the same as that given first above, only that the result is to be increased by *three* instead of *two*. But long before that time it is probable that the Gregorian calendar will be found itself to require correction.

4. To Find the Place (in the Calendar) of the Paschal Full Moon for any Year.

To FOUR TIMES *the Golden Number add the* CONSTANT FOR THE CENTURY. *If the Golden Number is even (but not otherwise), increase this sum by* FIFTEEN. *Then, if the result is* GREATER *than* TWENTY, *and less than* FIFTY, *it is the date of the Paschal full moon* CONSIDERED AS A DAY OF MARCH. *If it exceed* THIRTY-ONE, *subtract thirty-one, and the remainder is the date of Paschal full moon in April.*

If, however, the foregoing result be not greater than TWENTY, *and less than* FIFTY, *add* THIRTY *or subtract* THIRTY (*or* TWICE THIRTY, *if necessary*) *to bring it within these limits.*

Note that if the number obtained by this process be *exactly* twenty, or *exactly* fifty, it cannot be brought within the limits by the addition or subtraction of thirty. In this case the date of Paschal full moon is to be taken at *forty-nine*.

Note, also—and *particularly*—that if the number obtained by the rule be itself *forty-nine*, the Golden Number being, at the same time, *twelve*, or *more than twelve*, the date of Paschal full moon is to be taken at *forty-eight*.

Example.—What is the date of Paschal full moon in 1871?

The Golden Number, as just found, is *ten*, and the constant for the century is *ten* also. The Golden Number, moreover, is *even*. Then,

$$P = 10 \times 4 + 10 + 15 = 65. \quad \text{And } 65 - 30 = 35.$$

Paschal full moon is, therefore, the 35th day of March; that is, the 4th day of April.

What is the date of Paschal full moon in 2258?

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* This constant, for old style dates, is always *two*. The secular correction was introduced with the Gregorian calendar, the uncorrected constant being also increased from *two* to *nine*.

† These numbers diminished by *nine* will give those corresponding to the same centuries which are found in Table II. of the general tables at the end of the introduction to the Prayer Book.

By the rules foregoing, we find the Golden Number to be *seventeen*, and the constant for the century *twelve*. Then,

$$P = 17 \times 4 + 12 = 80. \quad \text{And } 80 - 30 = 50.*$$

The date of Paschal full moon, in this case, must be taken, according to note first above, at 49. It is, accordingly, the 49th of March, or the 18th of April.

What is the date of Paschal full moon in 3966?

The Golden Number is *fifteen*, and the constant for the century is *nineteen*. Then,

$$P = 19 + 4 \times 15 = 79. \quad \text{And } 79 - 30 = 49.$$

But because the Golden Number exceeds *eleven*, this must be taken, according to the second note foregoing, at 48. Paschal full moon is, therefore, the 48th of March, or the 17th of April.

5. To Find Easter Sunday.

To the constant number, EIGHTEEN, *add the value of the Sunday Letter, and afterward, if necessary, add* SEVEN, *or such* NUMBER OF SEVENS *as may suffice to make the sum greater than the date of Paschal full moon considered as a day of March, and no more. This sum is the date of Easter considered as a day of March. If it exceeds* THIRTY-ONE, *thirty-one is to be subtracted, and the remainder is the date of Easter in April.*

EXAMPLE.—What is the date of Easter in 1871?

We have found the Sunday Letter to be A = 1, and the date of Paschal full moon to be 35. Hence,

$E = 18 + 1 + 7 \times 3 =$ 40th day of March = 9th day of April.

6. Miscellaneous Rules for the Movable Feasts and Fasts of the Church.

For Septuagesima Sunday.—Take *four* from the date of Easter (in leap year *three*) and go back two months.

EXAMPLE.—In 1871, Easter is April 9.

April 9 — 4 is April 5, and Septuagesima Sunday is February 5.

For Ash-Wednesday.—Add *thirteen* to the date of Easter (*fourteen* in leap year), and go back two months.

NOTE.—If the sum exceed the number of days of the month in which Easter falls, consider it, nevertheless, a day of that month.

EXAMPLES.—In 1871, Easter is April 9.

April 9 + 13 = April 22; and Ash-Wednesday is February 22.

In 1868 (leap year), Easter was April 12.

April 12 + 14 = April 26; and Ash-Wednesday was February 26.

In 1869, Easter was March 28.

March 28 + 13 = 41; and Ash-Wednesday was January 41 = February 10.

For Trinity Sunday.—Take *five* from the date of Easter, and go forward two months.

EXAMPLE.—In 1871, Easter is April 9.

April 9 — 5 = April 4; and Trinity Sunday is June 4.

For Whitsun-Day.—Take *twelve* from the date of Easter, and go forward two months.

EXAMPLE.—In 1871, Easter is April 9 = March 40.

March 40 — 12 = March 28; and Whitsun-Day is May 28.

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* If, as in this example, the Golden Number is fifteen, or more than 15, the process may be simplified by dropping 15 before multiplying. Thus, 17 — 15 = 2. Then,

$$P = 2 \times 4 + 12 = 20. \quad \text{And } 20 + 30 = 50,$$

as in the text.

In the example next following the Golden Number is 15; and 15 — 15 = 0. Hence, when the Golden Number is 15, the constant for the century, or this constant increased or diminished by 30, gives the date of Paschal full moon directly. Thus, in the example above,

$$P = 19 + 30 = 49;$$

which, because 15 exceeds 11, must be put = 48.

For Ascension-Day.—Take *twenty-two* from the date of Easter, and go forward two months.

EXAMPLE.—Easter in 1871 = March 40.

March 40 — 22 = March 18; and Ascension-Day is May 18.

For Advent Sunday.—The First Sunday in Advent can never be earlier than the 27th November, nor later than the 3d December = 33d November. Hence,

To or from the date of Easter, considered as a day of March, add or subtract *sevens* till the result falls between the limits *twenty-six* and *thirty-four*, exclusive of both; and the result is the date of Advent Sunday, considered as a day of November.

EXAMPLE.—Easter in 1871 is March 40.

March 40 — 7 = March 33; and Advent Sunday is November 33 = December 3.

Otherwise.—To *twenty-five* (the date of Christmas-Day) add the value of the Sunday Letter (making A = 8 instead of 1, the other letters retaining their usual values) and the result is the date of Advent Sunday, considered as a day of November.

EXAMPLE.—The Sunday Letter in 1871 is A = 8; and 25 + 8 = 33d November = 3d December.

In 1870 the Sunday Letter was B = 2; and 25 + 2 = 27th November = Advent Sunday in 1870.

It is convenient to be able to fix some points in the series of Sundays after Trinity. The smallest number of Sundays that can follow Trinity Sunday is *twenty-two.*

For the twenty-second Sunday after Trinity, take *four* from the date of Easter, and go forward *seven* months.

EXAMPLE.—Easter Sunday in 1871 being April 9, we have

April 9 — 4 = April 5; and the twenty-second Sunday after Trinity is November 5.

The ninth Sunday after Trinity is found by taking *three* from the date of Easter, and advancing *four* months. Thus, in 1871, April 9 — 3 = April 6; and August 6 is the ninth Sunday after Trinity.

Hence, Trinity Sunday, the twenty-second Sunday after Trinity, and the ninth Sunday after Trinity, form this series, easily remembered:

Easter — 5 = Trinity Sunday.

Easter — 4 = twenty-second Sunday after Trinity.

Easter — 3 = ninth Sunday after Trinity.

The month to be supplied will be inferred from the numbers.

The fifth Sunday after Trinity has the same date as Easter, three months later, when Easter is in April; its date is one less when Easter is in March.

The fourteenth Sunday after Trinity has the date of Easter increased by *one,* five months later.*

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* The following will be found, perhaps, more curious than useful

The twenty-seventh Sunday after Trinity (which marks the largest number of Sundays which can happen between Trinity and Advent), when referred to November, always happens on the same day of the month as Easter Sunday referred to March. Thus Easter is the 40th day of March (the 9th of April), and the twenty-seventh Sunday after Trinity is the 40th day of November (the 10th day of December) in 1871. November is also the eighth month after March. If now we add 13 to 27, continuously, suppressing 35 whenever the sum exceeds that number, adding also 3 to the month whenever 13 is added to the weeks, and subtracting 8 from the months whenever 35 is subtracted from the weeks, a series of Sundays will be obtained, of which the date will be successively *one* less, *two* less, *three* less, and so on, than the date of Easter.

Also, if we subtract 13 from 27, continuously, adding 35 whenever the subtraction is otherwise impossible, at the same time subtracting 3 from the months for every 13 from the weeks, and adding 8 to the months for every 35 added to the weeks, we shall obtain a series of Sundays of which the dates are *one* greater, *two* greater, *three* greater, and so on, than the date of Easter. But it is to be noted in making these additions and subtractions, that if a result is at any time reached exceeding 29, and less than 35, this is to be disregarded, and 13 added or subtracted again, the double addition or subtraction still changing the date only one unit.

To find the Number of the Sundays after Trinity.—From the whole number of days in March and April united (*sixty-one*) take the date of Easter as a day of March, and divide this difference by *seven.* The quotient is the number of Sundays to be added to *twenty-two* (the minimum number) in order to obtain the whole number of Sundays after Trinity.

NOTE that, in April, the date may be taken directly from thirty.

EXAMPLE.—In 1871, Easter is April 9; and 30 − 9 = 21, which, divided by seven, gives *three.* 22 + 3 = 25, the number of Sundays between Trinity and Advent in 1871.

To find the Number of Sundays after Epiphany.—From the date of Easter, considered as a day in March, take *eleven* (*ten* in leap year), and divide the result by *seven.* The quotient is the number of Sundays between Epiphany and Septuagesima Sunday.

EXAMPLE.—In 1871, Easter is March 40.

40 − 11 = 29; and 29 ÷ 7 = 4, the number of Sundays after Epiphany in 1871.

In 1872 (leap year) Easter is March 31; and 31 − 10 = 21, which, divided by seven, gives *three.* Hence, there are three Sundays after Epiphany in 1872.

The foregoing rules are given without demonstration. It is proper, however, to present the reasons on which they are founded; and this I will endeavor to do as succinctly as possible.

I. As to the rule for the Golden Number. This is merely an arithmetical artifice for performing with facility, and without the use of the pencil, the operation prescribed for the same purpose in the first of the tables of the Prayer Book. The first year of our era is historically ascertained to have been the second of the lunar cycle. Hence, the year of our Lord, increased by *one* and divided by *nineteen* (the number of years in a complete cycle), will leave as a remainder the number (in the cycle still incomplete) of the year under consideration.

To divide by nineteen is troublesome. But if we consider that 100 contains *five* nineteens and five over, and 400 contains twenty nineteens and twenty over—that is to say, twenty-one nineteens and *one* over—we shall see that the cycle advances but one unit in 400 years. This gives the reason for dividing the *hundreds* by *four*, and taking the quotient as the first term of the result. Each hundred of the remainder still outstanding advances the cycle *five* places. Hence, the remainder is multiplied by *five* for the second term. The years of the incomplete century are easily resolved into twenties; and each twenty is obviously one complete cycle

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These rules will hold true for a series of dates extending from twelve days before Easter to twenty days after.

The following table embraces a portion of the dates which may be thus found. The letter E stands for the date of Easter, considered as a day of March. There are added the corresponding dates for the year 1872:

No. of Sunday.	Date of Sunday.	Months after March.	Corresponding Dates in 1872.
Twenty-sixth Sunday after Trinity	E − 7	8	Nov. 24.
Thirteenth " "	E − 6	5	Aug. 25.
Trinity Sunday	E − 5	2	May 26.
Twenty-second Sunday after Trinity	E − 4	7	Oct. 27.
Ninth " "	E − 3	4	July 28.
Eighteenth " "	E − 2	6	Sept. 29.
Fifth " "	E − 1	3	June 30.
Twenty-seventh " "	E	8	Nov. 31 = Dec. 1
Fourteenth " "	E + 1	5	Aug. 32 = Sept. 1
First " "	E + 2	2	May 33 = June 2
Twenty-third " "	E + 3	7	Oct. 34 = Nov. 3
Tenth " "	E + 4	4	July 35 = Aug. 4
Nineteenth " "	E + 5	6	Sept. 36 = Oct. 6
Sixth " "	E + 6	3	June 37 = July 7
Twenty-eighth " "	E + 7	8	Nov. 38 = Dec. 8

and *one* over. Each single year remaining advances the cycle *one* also. Thus the second part of the rule is explained. To the united results thus obtained we add a unit, on account of the year of the cycle which had elapsed at the beginning of the era. Inasmuch as one hundred complete cycles amount to 1900 years, it is obvious that we may, before proceeding, reject nineteen from the number of the centuries as often as it occurs. Thus, if the number of the given year is very large, as 8963, we may reject four times nineteen, or seventy-six, from eighty-nine, leaving thirteen centuries to operate upon. In this case, also, we may adopt the simple mode above given of dividing by nineteen; that is to say, we may divide first by twenty, and then add the quotient to the remainder. Thus:

$89 \div 20 = 4$, with 9 remainder; and $4 + 9 = 13$.

Then, $13 \div 4 = 3$, with 1 remainder; and $3 + 1 \times 5 = 8$. Also, $63 \div 20 = 3$, with 3 remainder; and $3 + 3 = 6$.

$8 + 6 + 1 = 15$, the Golden Number in the year 8963.

II. In regard to the rule for the Sunday Letter. Here the first object is to find a simple way of disposing of the centuries. The common year begins and ends on the same day of the week. In the succession of ordinary years, therefore, the Dominical Letter goes backward each year one place. A leap year sets it back two places. Accordingly, in one hundred Julian years, the letter goes back one hundred and twenty-five places. But, if from this number we suppress the even sevens, we shall have a remainder of *six* only, showing that in a Julian century the Dominical Letter goes back *six* places, which is equivalent to a forward movement of *one* place. In the Gregorian calendar three centuries out of four have but twenty-four leap years instead of twenty-five. In each of these centuries the retrograde movement is, therefore, one less than that just found; that is to say, is only *five*, which is equivalent to a forward movement of *two*. Hence, in every Gregorian quadricentennium there will be three centurial forward steps of two places each, and one forward step of one place; or, in all, an advance of seven places, completing the cycle of the letters.

To find now the actual value of the letter for a given centurial year, we take the fact, historically ascertained, that in England, in 1752, after the intercalation in February (which intercalation really took place in 1751 in old style, as the year 1752 only began in Great Britain on the 25th March), the Dominical Letter was D. By the adoption, in that year, of the Gregorian calendar, the fourteenth day of September (which regularly fell on Monday) was put in place of the third of the same month (which was a Thursday), the day of the month being removed backward a week and four days. The letters during the remainder of the year fell, therefore, as if all the days of the year preceding this change (including the first of January) had been removed, in like manner, backward four days in the week. And as January 1st fell on Wednesday (since the Sunday Letter was E before the intercalation of February), the removal of this day backward four places would have carried it to Saturday. In other words, the effect of the suppression of eleven days in the calendar changed the Dominical Letters of the year from E and D to B and A.* Between 1752 and

* Valuable, and even necessary, as was the improvement made upon the Julian calendar by Pope Gregory XIII., it is impossible, at the present day, to regard without astonishment the extraordinary and totally unnecessary interference with systematic chronology caused by his arbitrary obliteration of ten days out of the month of October in the year 1582. Julius Cæsar had reason for his "year of confusion;" for the calendar was then out of joint with the year by nearly three months. Moreover, his momentary disturbance of the order of things was only a more signal but final example of the chronic confusion which, through the agency of the pontifical jugglers, had reigned before. Neither of these reasons existed in sufficient force to justify Pope Gregory in the introduction of his new year of confusion in 1582. The second did not exist at all; for the Julian calendar had secured a perfectly unbroken uniformity in the method of computing time for more than sixteen centuries. And the displacement of the seasons by the error of the Julian intercalation had been too trivial to occasion the slightest inconvenience, or to call for correction. What *was* desirable and all that was desirable, was to prevent any further displacement for the future; and this was accomplished by the simple and easily intelligible correction of the intercalation introduced by the mathematicians of Pope Gregory.

the close of the century, there elapsed forty-eight years, of which only eleven were leap years (the centurial years 1700, 1800, and 1900 being made common years by the Gregorian reformation). The Sunday Letter, therefore, went back fifty-nine places during this period; or, suppressing the sevens, *three* places only, which is equivalent to advancing *four* places. And four places added to 1 (= A) carries us to 5 (= E), which was the value of the Dominical Letter in 1800. In 1900 the value will be found to have advanced *two* places more, so that it will have become 7 (= G). But in 2000 (a leap year) there will have occurred an advance of only *one* additional place, giving a value of 8. And $8 - 7 = 1 = A$, which is always the Sunday Letter of the bissextile centurials. Thus, starting with the centurial year divisible by 400, we have the successive centurial years of each quadricentennium marked by the series, 1, 3, 5, 7; or A, C, E, G.

In dividing the incomplete century into *twenties* and *fours*, the object is to obtain small numbers to operate on, and numbers into which the entire number is easily resolved. Every twenty years, from the beginning of the century, must contain five leap years (except, three fourths of the time, the *last* twenty; but with this last one we have nothing at present to do). In every twenty years, therefore, the Sunday Letter moves backward twenty-five places; which, suppressing sevens, is four places; being equivalent to a forward movement of *three* places.

In every additional *four* years the letter moves backward *five* places, which is equivalent to a forward movement of *two* places.

And in every additional *single* year the letter moves backward one place, which is equivalent to a forward movement of *six* places. These outstanding single years can never be leap years, as is obvious from the fact that the last year of every twenty, and the last year of every four, previously taken, is a leap year.

The reason of the rule for the Dominical Letter is thus made obvious. After a little familiarity with its use, it will be found more convenient, in dealing with the *residual* years, to *subtract* their number (it can never exceed *three*) from the numbers previously found, instead of adding six times their number; or, otherwise, to subtract their number from *seven*, and add the difference. In the first instance, however, it facilitates recollection, as it also gives uniformity of character to the rule throughout, to make the terms all

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The real reason for the suppression of the ten days was not that society was suffering or astronomy embarrassed in consequence of the inconsiderable retrogradation of the equinoxes in March and September which had taken place during the lapse of several centuries. It was that, in 1575, the vernal equinox fell on the 11th of March, whereas, in 325, at the time of the assembling of the Council of Nice, which put an end to the differences previously existing between the East and the West in regard to the time of celebrating the festival of Easter, it was supposed to have fallen on the 21st. That the restoration of this coincidence was of no practical importance, even in an ecclesiastical point of view, is manifest from the fact that, in the authorized *Explicatio Romani Calendarii*, etc., published at Rome by the Pope's principal mathematician, Clavius, in 1603, the author takes pains to insist that the Church is under no obligation to make Easter a movable feast at all; but that she does so merely out of respect to an ancient custom. Considering this fact, it is the more surprising that the extraordinary interruption in the succession of the days of the year introduced along with the new calendar should have been admitted at all, since it was by no means done without reflection, nor without extensive consultation with civil as well as with ecclesiastical rulers. As early as 1577, five years before the appearance of the Papal decree declaring the change, the whole scheme was submitted to all the Roman Catholic princes in Europe; and it not only elicited no objection, but failed to provoke a remark of even doubtful approval. On the other hand, according to Montucla, it was everywhere eulogized in the most unqualified terms.

Had it not been for this feature of the proposed reform, the old mode of computing the lapse of time would have passed into the new without any sensible dislocation at any point of the record. And whether or not it had been immediately received by peoples not in communion with the Church of Rome, it could have introduced no difference into the *civil* calendars of the East and the West, or of Romanists and Protestants, until at least the lapse of nearly a century and a quarter (the year 1700), long before which time it would probably have been universally accepted upon its own merits and without regard to the manner of its origin. These ten days undoubtedly prevented its acceptance in Protestant England for nearly two hundred years (when the difference had become eleven days), and they still keep the great empire of Russia out of harmony in regard to this matter with the rest of Europe and with all America, the difference being now twelve days.

immediately additive. They will all be small, and the process by which they are obtained may soon be so mastered as to make the determination of the Sunday Letter for any year in the past or the future a problem requiring but a moment's thought, without the use of any implements of calculation.

In regard to the alternative rule, after what has been said above the explanation will be obvious. For every four years ending with a leap year, the Sunday Letter advances two places. By dividing the year from the beginning of the century by *four*, and multiplying the quotient by *two*, therefore, we shall have the total advance of the letter at the last leap year preceding the year given; or in the given year itself, if that is a leap year. But this division by four and multiplication by two gives a result which may be more directly obtained by simply dividing by two at once. Moreover, to add the complement of the residual is equivalent to subtracting the residual itself. Hence, it is seen that the alternative rule rests on the same principles precisely as the rule first given.

The method may be applied to the finding of the Sunday Letters in the period when the old style of reckoning still prevailed, if we consider that, as all the centurial years were then leap years, the movement of the letter from one centurial year to another was always one place forward. Accordingly, after every seven centuries, the centurial year encounters the same letter a second time. It remains to ascertain the value of this letter for some known centurial year. We have seen that, in 1752, after the intercalation, the Sunday Letter was D. In the fifty-two years which had elapsed since 1700, the Sunday Letter had gone back sixty-five places. Therefore, if we descend in time from 1752 to 1700, the letter should *go forward* sixty-five places from D (or 4), which we find to be the place in the year first mentioned.

$65 + 4 = 69$; and $69 \div 7 = 9$, and 6 remainder.

Thus, the value of the Sunday Letter was $6 = F$, in the centurial year 1700, after the intercalation of that year. As by the Julian reckoning the letter advancs one place per century, it follows that if we go back seventeen places from $F = 6$, we shall find the letter corresponding to the year *Zero*, that is, to the year last preceding our era. Seventeen, with the sevens suppressed, becomes *three*, which, taken from *six*, leaves $3 = C$ for the Sunday Letter of the year 0. In the year *one*, therefore, the letter was $3 - 1 = 2 = B$; or, in other words, the era commenced on a Saturday.*

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* This result is obtained by computing backward, as above, to the beginning of the era, according to the Julian calendar as in use in the sixth century, and since, in the Western Churches. But according to Blondel (*Histoire du Calendrier*, cited by Delambre) the differences between the Julian calendars of Alexandria and Rome occasioned long dispntes, which were threatening to lead to violence, when happily the parties were pacified by the successful efforts of Dionisius Exiguus, a monk of Rome, who persuaded the Western Christians to adopt the Alexandrian calendar. What probably contributed largely to his success was his proposition that all Christians should unite in referring dates to the year of our Saviour's birth. Before this time, 'says Delambre (*Histoire de l'Astronomie Moderne*, vol. i.), there had been, in this respect, no uniformity of usage, some counting the years from the era of Diocletian, which they called also the Era of the Martyrs; some from the day of the Passion; some, like the Romans, from the foundation of Rome; and others designating years by the names of the consuls or emperors.

The first year of the Julian calendar, according to Montucla, Delambre, and other authorities, corresponded to the forty-sixth before our era. By a misunderstanding on the part of the priests, to whom was committed the charge of the calendar, the intercalation was made twelve times successively at the end of every third year, instead of every four; the period of four years being made out, according to a method of reckoning common among the Romans and in the East, by counting the bissextile year at the beginning as well as that at the end. There had then been introduced three intercalary days too many; and to correct the error thus occasioned, it was ordered by Augustus that the intercalation should be omitted during the twelve years beginning with the thirty-seventh of the era, and ending with the forty-eighth. If these numbers are to be depended upon, it would seem that, in the settlement of the controversy among the early Christians in regard to the Julian calendar, the regularity of the succession of bissextile years, in the order in which it was established by its founder, was interrupted: a not unnatural consequence of the yielding of Rome (where the true order was most likely to be preserved), for the sake of peace, to Alexandria. However this may be, it is evident that any interpretation of the figures will make it probable that one intercalation, and only one, was suppressed subsequently to the commencement of our era. The effect of this, if it occurred, was to move backward the

The years 100, 200, 300, etc., were marked by letters advanced *one* place, *two* places, *three* places, and so on, beyond C; that is to say, their letters may be found by adding *three* to the number of the century. If the number exceed *seven,* seven must, of course, be suppressed. Hence, for years reckoned according to the *old style,* the Sunday Letters are found in the same way as for Gregorian years, except that the *centurial* term is not to be taken from the series heretofore given, viz, 1, 3, 5, 7, but ascertained by adding three to the number of the century, and suppressing sevens.

III. In order to explain the rule for the secular correction, and for the Paschal full moon (IV.), it is necessary first to say a word in regard to the relation between the lunar month and the solar year, on which the correction depends. The periods of revolution of the earth and moon around their respective centres of motion are not commensurable; nor are they connected by any approximate numerical relation which is at all obvious. The discovery, therefore, which was made in the fifth century before our era, that there is a period at the end of which these two bodies return to their original positions relatively to the sun, with a very inconsiderable difference, was a very interesting, and in some respects important, incident in the history of astronomy. This period is nineteen years, and the nearness of approach to coincidence of the two movements at its close will be understood from the following statements:

The length of the true solar (tropical) year is 365 days, 5 hours, 48 minutes, 46.05444 seconds; or, expressed in days and decimals, 365.2421997. The length of a mean lunation is 29 days, 12 hours, 44 minutes, 2.84 seconds; or 29.530588259 days. Hence, nineteen tropical years contain 6939.6017943 days, and two hundred and thirty-five mean lunations, 6939.6882801 days. The difference is 0.0864858 days, amounting to a little more than two hours four and a half minutes, by which the lunar period is in excess. Supposing, therefore, that at the commencement of a given year the moon is at a certain determinate point of advancement in the lunation, then at the beginning of the twentieth year following (or the end of the nineteenth) it will not be quite so far advanced; or, in common language, its *age* will be less. The age of the moon at the beginning of the year is what is called its *epact;* and it thus appears that the effect of the inequality above spoken of is to occasion a slow diminution of the moon's epact from cycle to cycle. It is this slow diminution which necessitates the application, to the constant employed in finding the date of the Paschal full moon, of the *secular correction* which we are considering. As the epact diminishes, the time of a particular phase of the moon (as, for instance, the new or the full moon) is retarded in its occurrence, or pushed forward to a later period in the month. The retardation amounts to a day in about two hundred and twenty years (219.7), and to thirty days, or the length of an entire lunation, in 6,600 years. In the second of the general tables relating to the calendar in the Prayer Book, are found the values of the necessary corrections corresponding to the successive centuries, as computed in the sixteenth century by the mathematicians of Pope Gregory XIII. Sixty-six hundred years added to the fifteen centuries preceding the Gregorian reform, carry us to the eighty-first century. The correction in the table is twenty-eight, instead of thirty. This arises from the fact that the length of the tropical year, and of the mean lunar month, as now received, differ slightly from those employed in the original calculation of the table. According to Clavius, the authorized expositor of the reformed calendar, the diminution of the moon's epact amounted to only 0.080825237 days per cycle; at which rate a retardation to the extent of an entire lunation would require the lapse of something over seven thousand years. Seven thousand added to fifteen hundred gives 8,500, opposite to which we find in the table

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Dominical Letter of the first year of the era a single place, or from B to A; so that, in point of fact, this year began on Sunday instead of on Saturday, as it is computed to have done above, and as it would have done had the intercalations from the beginning been regularly made.

the correction 30, or 0. The error of Clavius, if corrected, would produce no difference in the tabular numbers before the year 4800, and therefore need not concern us now.

An inspection of the numbers in the table will show that they do not increase uniformly, so as to keep pace with the uniform diminution of the epact. The reason of this is, that, for practical purposes, it is necessary that the civil year should be made to consist of a determinate number of entire days; and, therefore, as the astronomical year contains a fraction of a day, the civil years cannot all be equal in length. It is also convenient to make the *lunar months*, in the calendar, to consist of entire days, which renders it similarly necessary to make the successive months unequal. The several steps by which Clavius effected the adjustment of the lunar to the solar period were the following:

As a first approximation, the solar year was taken at 365 days exactly. A lunar year was then assumed of twelve months, consisting alternately of thirty-days and of twenty-nine days each. This gives, in effect, a mean lunar month of twenty-nine and a half days (which is about three quarters of an hour too short), and a lunar year of 354 days, less than the solar year by eleven days. If a calendar new moon occurs, therefore, at the beginning of a year exactly, it is obvious that, at the beginning of the next year, there will be a calendar moon eleven days old. In other words, the epact of the moon is annually increased eleven days. Supposing the original epact to be Zero, there will then be, in three years, an epact of thirty-three days. As this exceeds a lunation, a lunation may be dropped, and the epact correspondingly reduced. After three years more it will be necessary to do the same thing again; and in the course of a cycle this necessity will arise seven times. The lunations thus superadded to the regular lunar months are called *embolismic*, which term is expressive of superaddition. The embolismic months are all made to consist of thirty days, except the last one of the cycle, which has but twenty-nine days. The reason of this will appear from the following comparisons, which are given for a period of four complete cycles, or seventy-six years:

76 years of 365 days each, contain..................................	27,740.000 days.
940 true mean lunations "	27,758.753 "
Difference.........................	18.753 "

These $18\frac{3}{4}$ days are a little more than balanced by the nineteen intercalary days belonging to the *leap years* in the four lunar cycles, of which no account has yet been taken. But 940 lunations, taken at twenty-nine and a half days each, amount to only 27,730 days, or fall short by ten days of the number found in seventy-six common years. As there are twenty-eight embolismic months in the four cycles, it is possible to dispose of these ten days by adding half a day each to *twenty* out of the number, making them thus to consist of thirty instead of twenty-nine and a half each. There remain, then, eight months of mean length, four of which may be reduced to twenty-nine days, while the other four are made thirty. Of the twenty-eight embolismic months, therefore, twenty-four, or six in each cycle, will have thirty days; and four, or one in each cycle, will have twenty-nine days. For convenience, this short embolismic month is put last.

But we see now that the intercalation has put the solar period in excess by about a quarter of a day in seventy-six years; to that extent increasing the moon's epact. In four times 76 years, that is, in 304 years, this increase will amount to a day. If the more exact numbers be taken, it will appear that nearly 308 years are necessary to increase the epact an entire day. The numbers employed by Clavius gave $312\frac{1}{2}$ years. This is equivalent to eight days in 2,500 years. He proposed, therefore, to increase the epact by a day at the end of every 300 years, seven times successively; and to make the addition of an eighth day at the end of the following 400 years. This is the method of making the lunar correction actually employed in the Gregorian calendar. But this correction adjusts the place of the moon *in*

the Julian year. The error of the Julian year was provided for in the new calendar, by a correction also taking effect in passing from century to century. The excess of the true year above 365 days is not quite a quarter of a day, but a fraction expressed by the decimal 0.2421997. The Julian calendar adds one day to every fourth year, or 100 days in 400 years. But if we multiply the foregoing fraction by 400, the result will give us 96.87988 days,—that is, not quite 97. The Julian calendar, therefore, adds about three days too many in every four hundred years. The Gregorian correction consisted, accordingly, in reducing three leap years in every four centuries to common years: and for convenience and facility of recollecting the correction, the years chosen to be so reduced were the final years of those centuries which are not divisible by four; those which *are* so divisible continuing to be leap years. It is evident from the numbers above given that this correction is not quite adequate. There remains outstanding a minute error, which may amount to a day in something more than 3,300 years. Clavius supposed it to be greatly less than this, and regarded it, therefore, as too unimportant to require attention. At least, he proposed to leave it to posterity to look after.

As this solar correction makes the year begin sooner than it otherwise would, it reduces the age of the moon, or the epact, by the same amount. The lunar and solar corrections are, accordingly, opposed to each other; but the solar, being the greater on the whole, prevails, and the epact gradually diminishes. As the lunar correction is applied once in three centuries, and the solar three times in every four, the two corrections may occasionally come together; in which case the epact stands over unaltered from one century to the next. If the lunar correction falls at the end of a quadricentennium, the epact goes a unit forward. This would happen regularly once in 1200 years, and no oftener, if the interval between the successive lunar corrections were always three centuries, and no more. But as the correction stands over to the fourth once in twenty-five centuries, the recurrence of this advance is not quite regular. Usually, when there is a change, the movement is retrograde. The numbers in the General Table II., in the Prayer Book, are increasing; but, as related to the epact, they are subtractive.

From what has been said, it will be manifest that the moon of the ecclesiastical calendar has very little to do with the actual moon in the heavens; agreeing with that only in the length of its mean period. It was, in fact, a part of the deliberate design of the contriver that the ecclesiastical moon should always fall at least a day later in the month (although it does not always happen so) than the true moon, in order that the festival of Easter might not fall upon the same day on which the Jews celebrated their Passover, or the Quartadeciman Christians their Easter. He claimed for it, in fact, no astronomical significance, but called it what it is, simply a cycle. His manner of indicating epacts in the actual calendar of the year shows his independent disregard of the real moon. In the first place, the days of the year are divided off into twelve lunar months of thirty and twenty-nine days each, alternately; the divisions falling as it may happen among the days of the ordinary calendar months. The end of the lunar year thus falls on the 20th of December. No modification of this mode of division is made on account of the additional day of leap year; the 29th of February being counted with the 28th as one day.* The eleven days outstanding at the end of the lunar year constitute the increase of the epact in passing to the year following. Next, it is assumed that, whatever may be the age of the moon on the 1st of January, the same will be true of the successive moons throughout the year, at the beginning of the successive lunar months. And

* This is in accordance with modern municipal law. But, in point of fact, in the Church Calendar, it was the 25th, which was counted as one day with the 24th; this latter being the *dies sextus ante Kalendas Martias*, and the leap year 25th, that which was later known as the *dies bissextilis ante Kal. Mar.*, which gave name to the year.

as, by counting backward from January 1st into December a number of days equal to the epact, we shall come to the place of the new moon next preceding January 1st, so by counting back from the first day of the second lunar month (which is January 31st) we shall find the place of the new moon in January, and so on. In order to save the trouble of counting, and to enable a person to find the place of new moon from the epact by inspection, the contriver of the calendar marked in, in Roman numerals, the epacts corresponding to all the days of new moon possible in a month; these numbers forming a series descending from *thirty* to *one; thirty* standing opposite the first day, and *one* opposite the last day of the lunar month.

In the application of this method there arises an embarrassment, growing out of the fact that, since the alternate lunar months have only twenty-nine days, they have not places enough for thirty epacts. This difficulty is surmounted by the expedient of writing, in the calendar of these months, *two consecutive epacts* opposite to *one* day, which is equivalent to bringing, occasionally, in the short months (or, as they were called, the *hollow* months) of the same cycle, two new moons to the same day, when they fell on distinct days in the long or *full* months. The epacts XXIV and XXV were selected for this duplication, the reason of which choice will presently appear. In the following table is presented the arrangement of the calendar according to Clavius, for the first four months of the year, which, as it extends beyond the latest date of Easter, is sufficient for our present purpose:

Day of Month.	January.		Day of Month.	February.		Day of Month.	March.		Day of Month.	April.	
	Epact.	Letter.		Epact.	Letter.		Epact.	Letter.		Epact.	Letter.
1	0 or XXX.	A	1	XXIX.	D	1	0 or XXX.	D	1	XXIX.	G
2	XXIX.	B	2	XXVIII.	E	2	XXIX.	E	2	XXVIII.	A
3	XXVIII.	C	3	XXVII.	F	3	XXVIII.	F	3	XXVII.	B
4	XXVII.	D	4	XXVI., 25	G	4	XXVII.	G	4	XXVI., 25	C
5	XXVI.	E	5	XXIV,XXV	A	5	XXVI.	A	5	XXIV,XXV	D
6	XXV., 25	F	6	XXIII.	B	6	XXV., 25	B	6	XXIII.	E
7	XXIV.	G	7	XXII.	C	7	XXIV.	C	7	XXII.	F
8	XXIII.	A	8	XXI.	D	8	XXIII.	D	8	XXI.	G
9	XXII.	B	9	XX.	E	9	XXII.	E	9	XX.	A
10	XXI.	C	10	XIX.	F	10	XXI.	F	10	XIX.	B
11	XX.	D	11	XVIII.	G	11	XX.	G	11	XVIII.	C
12	XIX.	E	12	XVII.	A	12	XIX.	A	12	XVII.	D
13	XVIII.	F	13	XVI.	B	13	XVIII.	B	13	XVI.	E
14	XVII.	G	14	XV.	C	14	XVII.	C	14	XV.	F
15	XVI.	A	15	XIV.	D	15	XVI.	D	15	XIV.	G
16	XV.	B	16	XIII.	E	16	XV.	E	16	XIII.	A
17	XIV.	C	17	XII.	F	17	XIV.	F	17	XII.	B
18	XIII.	D	18	XI.	G	18	XIII.	G	18	XI.	C
19	XII.	E	19	X.	A	19	XII.	A	19	X.	D
20	XI.	F	20	IX.	B	20	XI.	B	20	IX.	E
21	X.	G	21	VIII.	C	21	X.	C	21	VIII.	F
22	IX.	A	22	VII.	D	22	IX.	D	22	VII.	G
23	VIII.	B	23	VI.	E	23	VIII.	E	23	VI.	A
24	VII.	C	24	V.	F	24	VII.	F	24	V.	B
25	VI.	D	25	IV.	G	25	VI.	G	25	IV.	C
26	V.	E	26	III.	A	26	V.	A	26	III.	D
27	IV.	F	27	II.	B	27	IV.	B	27	II.	E
28	III.	G	28	I.	C	28	III.	C	28	I.	F
29	II.	A				29	II.	D	29	0 or XXX.	G
30	I.	B				30	I.	E	30	XXIX.	A
31	0 or XXX.	C				31	0 or XXX.	F			

The conditions of the problem in regard to Easter are these: Easter must be a Sunday, and it must fall later than the full moon which happens on or next following the vernal equinox. But the words *full moon* are not to be understood to mean the astronomical full moon, either true or mean; but the fourteenth day of an ecclesiastical lunation, of which lunation the

degree of advancement is expressed by the epact of the year. In this count of fourteen days, the day of the new moon is itself included; so that the date of full moon, so far as concerns the determination of Easter, will be ascertained by deducting thirteen from the epact of the year (adding *thirty*, if necessary, to make the subtraction possible), and entering the calendar with the result. Thus, the epact of 1871 is nine. From this, increased by thirty, if we deduct thirteen, there remains twenty-six, and this number in the calendar stands opposite the 5th day of March, and the 4th day of April. The first of these dates, falling before the equinox (March 21), is rejected, and the other, April 4th, is taken as the date of Paschal full moon. Now, as the Sunday Letter of the year is A, we look down the column of letters after April 4 until we come to A, which stands opposite April 9, and determines that to be the date of Easter for 1871.

It was supposed by Clavius, in accordance with the astronomical knowledge of his time, that the vernal equinox would fall generally on the 20th, and never later than the 21st of March. This, therefore, determined the date of the earliest possible Easter full moon. The corresponding new moon, being the fourteenth day earlier, or the 21st less thirteen days, is brought back to the 8th of March, opposite to which in the table stands the epact XXIII. For any larger epact, as XXIV, the Easter new moon must be looked for a month later, or in April. Since, therefore, the epact XXIII gives the earliest new moon on which Easter can depend, so the epact XXIV gives the latest. And this appears to have been the reason determining the author of the calendar to bring together, in the hollow months, the two epacts XXIV and XXV, rather than any two others, opposite to the same day. The effect of this is to make a lunation commencing on the 7th of March to consist of twenty-nine days only, while all lunations commencing on the days of March earlier than the 7th consist of thirty days. It also makes it possible that two new moons in the same cycle may occur on the same day of April, viz., the 5th; and this is an occurrence against which, however unimportant it may appear, Clavius seemed to think he ought to guard. He therefore resorted to an expedient which will be understood by considering the epacts of the successive years of the cycle as they stood in the century in which he lived, and subsequently to the reformation of the calendar. They are as follows, the numbers denoting the years of the cycle being placed over them severally:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
I.	XII.	XXIII.	IV.	XV.	XXVI.	VII.	XVIII.	XXIX.	X.	XXI.
12.	13.	14.	15.	16.	17.	18.	19.			
II.	XIII.	XXIV.	V.	XVI.	XXVII.	VIII.	XIX.			

The epacts in the second line exceed those above them in the first, by a single unit in each case. The *year number* in the second line exceeds that in the first, in each case, by *eleven*. We perceive the reason of this law when we consider that, in eleven years, the epact is increased eleven times successively by eleven each time, or by 121 in all; and that out of this increase four embolismic months of thirty days each are dropped, leaving a remainder of only one. In the whole series of nineteen epacts there can, accordingly, be found eight *pairs* of numbers differing in each case by a unit; but there cannot be found among all the nineteen so many as *three* numbers which, put together, will form a continuous series with only a single unit for the common difference. And though, from century to century, these epacts are gradually diminished by the application of the secular correction, yet they are all diminished equally, and their respective differences remain unchanged. It will happen, therefore, that if at any time both XXIV and XXV are in the series, XXVI cannot be in it at the same time. By adding, for example, a unit to all the epacts given above, XXIV and XXV would belong to the 3d and 14th years; but XXVI, which stands under 6 at present, would become XXVII. Or, by subtracting two units from all the epacts, XXIV and XXV would belong to 6 and 17, while XXIV, under 14,

would become XXII. This state of things will exist after the year 1900. The epacts of the present century are those of the foregoing series reduced by *one.* We have XXV and XXVI in the 6th and 17th years of the cycle, but XXIV does not occur at all.

To meet the case in which XXIV and XXV are both present, it was provided by Clavius that XXV should take, for the period over which this state of things extends, the vacant place of XXVI; and this explains the introduction of the 25 in Arabic numerals, which stands by the side of XXVI in the table. The 25 is introduced into the full months as well as into the hollow months, though quite unnecessarily. By this contrivance, a separate day of the month is provided for every Paschal full moon.

The practical advantage gained by this elaborate contrivance is exceedingly small. It prevents the possible occurrence of Easter on the same day of the month twice in the same cycle, when the Sunday letter happens to be C. It will be observed, from an inspection of the series of epacts just given, that if there are two in the cycle differing by a unit, the larger will arrive eleven years after the less. If XXIV and XXV are both present, therefore, the Golden Number corresponding to XXV must be greater than eleven. Eleven consecutive years will, three times out of four, embrace three leap years; so that the Sunday Letter will move in this time, fourteen places; or, in other words, two years distant from each other by an interval of eleven, will have the same Sunday Letter. If the epact is XXV, the corresponding Paschal full moon will fall on the 18th day of April, to which, as the table shows, the letter C belongs. If, therefore, C is the Sunday Letter, Easter will fall on the 25th April, which is the Sunday following. But the same will be true if the epact is XXIV. By changing XXV, however, to 25—that is, to XXVI—the Paschal full moon for this epact falls on Saturday, April 17th, and Easter is the next day, April 18th, while the epact XXIV still gives the 25th for Easter. It is doubtful whether the object secured by this elaborate contrivance was worth the trouble, especially when it is considered that Easter often unavoidably falls on the same day of the month within the same cycle, and is always liable to do so with certain Sunday Letters in years differing by an interval of *five* or *six* or *eleven.** Years differing by five have the same Sunday Letter if two leap years intervene; and years differing by six similarly agree if but one leap year falls in the interval. Now, the years 1 and 7 of the cycle differ by 6, and have epacts differing by 6; and the years 2 and 7 differ by 5, and have epacts differing by 5. The same is true of other numbers of the series. There is, accordingly, always a liability to the recurrence of Easter on the same day, when the earliest of the two paschal full moons falls on Sunday or Monday. The chance of coincidence is much greater when years differ by eleven, since their epacts differ by only one.

The duplication of epacts on the 5th of April has, as above remarked, the effect to make the distance between successive new moons only twenty-nine days in the limiting case; while all those corresponding to greater epacts are distant by thirty. This occasions a little irregularity, which might have been avoided by making the duplication on a day later in the month; in which case it would have affected no lunation regulating Easter. The plan

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* Thus, during the current cycle Easter occurs on the same day in the following years differing by *eleven*, viz.:

In 1863 and 1874..On the 5th of April.
In 1865 and 1876..On the 16th of April.
In 1866 and 1877..On the 1st of April.
In 1867 and 1878..On the 21st of April.
In 1869 and 1880..On the 28th of March.

In the following differing by *six*, viz.:

In 1869 and 1875..On the 28th of March.
In 1873 and 1879..On the 13th of April.

And in the following differing by *five*, viz.:

In 1875 and 1880..On the 28th of March.

here suggested would have been attended with no practical consequence except that, in one combination of circumstances, it would have thrown Easter forward from the 19th to the 26th of April; a matter of no moment, considering that the festival already occurs as late as the 25th. It would, doubtless, have been a great advantage had Easter been restricted to a much more limited range of movement, and not been tied to the moon at all; but the saving of a day where the range still extends over thirty-five days, is an insignificant benefit.

From what has been said, the break in the third of the general tables relating to Easter in the Prayer Book will be understood. We have seen that when the epact is XXV, and the year in the cycle is greater than 11, XXVI is taken instead of XXV, making April 4th the day of new moon, instead of April 5th; and April 17th the day of Paschal full moon, instead of April 18th. The break in the table occurs between the eleventh and twelfth years of the cycle, and all the later numbers stand opposite to April 17th.

It has been stated that in the sixteenth century, after the reformation of the calendar had taken place, the epact 1 fell on the first year of the cycle. As the reformation, by the suppression of ten calendar days, had reduced all the epacts to a corresponding extent, it would appear that, previously to this time, the epact of the first year of the cycle had been 11. In point of fact, it was 8. Apparently it would have answered every practical purpose if the author of the new calendar had been content to take this fact as he found it. He thought proper, however, to make two or three arbitrary assumptions in order to connect his work with a much earlier epoch. As the lunar correction is governed by periods of twenty-five centuries, he took the year 1800 as the end of one of these periods. He assumed, accordingly, that the correction had been regularly applied in the years 1400, 1100, and 800.* He further assumed that, previously to the year 800, the epact was zero in the third year of the cycle (making it 8 in the first), and that this had been the case during the previous centuries, from the time of the council at Nicæa. The application of the lunar correction in the year 800 was then supposed to increase the epact in the first year of the cycle to 9. A similar correction in 1100 was supposed again to make it 10; and a third in 1400 to make it 11; after which the suppression of the ten days, as above stated, reduced it to 1. By this artifice, the change in the epact made in 1582 was really only seven, while it was nominally ten.

When the epact in the first year or last year of the cycle is known, that of any other year of the cycle may easily be calculated. As the increase, from the first year onward, is eleven days per annum, we have only to add to the epact of the first year, *eleven*, multiplied by the number of the year in the cycle, *less one*. Thus, if ε' is the epact of the first year, and ε that of the nth year, we shall have

$$\varepsilon = \varepsilon' + (n - 1) \times 11 = \varepsilon' + n + 10\,(n - 1) - 1.$$

Of course, if the result exceeds thirty, thirty, being an embolismic month, must be dropped. Putting $\varepsilon' = 1$, as was true at the formation of the calendar, we shall have the simple formula,

$$\varepsilon = n + 10\,(n - 1),$$

or the epact is equal to the number of the year in the cycle, increased by ten times the number next less. The value of ε', however, became zero in 1700, and since that time the proper formula has been

$$\varepsilon = 11\,(n - 1),$$

or the epact is equal to eleven times the number next below that which ex-

* To have completely eliminated the preëxisting error of the epact, the imaginary correction should have been applied also in the year 500, the ecclesiastical moon having been behind the astronomical mean moon more than four days. It was not thought advisable to do this, because it would have put the ecclesiastical moon, for half the time, *earlier* than the *true* moon, according to which latter the Jews regulated their month Nisan, by direct observation. By keeping the calendar moon a day behind the astronomical mean moon, Clavius believed that he had effectually provided against the possibility of the occurrence of Easter before the fourteenth of Nisan, or the Jewish Passover, in any year; or before the true full moon of the Paschal month.

presses the place of the given year in the cycle. Or the former rule can be used, and the result diminished by one. In 1900 and after, the former rule may still be used, the result being diminished by *two*.

If ε'' be used to express the epact of the *last* year of the cycle, the value of ε will be found by adding to ε'', in the first place, twelve, which is the increase of the epact in passing from one cycle to another, and eleven for each succeeding year. Thus:

$$\varepsilon = \varepsilon'' + 12 + 11\,(n-1) = \varepsilon'' + 11\,n + 1.$$

Thus, in the latter part of the sixteenth century, ε'' was 19. For the first year of the cycle, we shall find, putting $n = 1$,

$$\varepsilon = 19 + 11 + 1 = 31\text{; or, dropping 30, } \varepsilon = 1.$$

At present, $\varepsilon'' = 18$. Hence, for the present year, 1871, which is the tenth year of the cycle,

$$\varepsilon = 18 + 11 \times 10 + 1 = 129\text{; excluding 30s, } \varepsilon = 9$$

The equation, $\varepsilon = \varepsilon'' + 11\,n + 1$, may be put in this case:

$$\varepsilon = 18 + 11 + 11\,(n-1) + 1 = 11\,(n-1) + 30 = 11\,(n-1),$$

which result agrees with that obtained before.

When the epact is known, we find the day of new moon in March by looking into the ecclesiastical calendar given above, where this day stands opposite the given epact. To find the day of full moon, we add *thirteen*, which carries us forward to the *fourteenth day of the moon*, so called. If the full moon happens earlier than the 21st of March, it cannot be the Paschal moon. In that case, we must, accordingly, look in April.

A mode of proceeding which dispenses with the table is the following: The earliest day of March on which the Paschal full moon can fall is the 21st. The new moon which corresponds to this happens on the 8th ($21 - 13 = 8$), and the epact corresponding to the 8th is 23. Now, if the epact 23 gives full moon the 21st, the epact 22 will give full moon the 22d, the epact 21, full moon the 23d, and so on. In other words, as the epact diminishes, the date of full moon equally increases; and the sum of this date and the epact leading to it is a constant sum. This constant is, of course, equal to $21 + 23$, or 44.

The following rule, therefore, is general: *Subtract the epact from 44, and the remainder is the date of full moon considered as a day of March.*

But if this remainder is less than 21, the full moon so found cannot be the Paschal full moon. In this case, therefore, we must increase the constant 44 by 30 (the number of days intervening, as we have seen above, between the new moons of the early days of March and those which follow them in April), giving us a new constant, 74; and from this, if, as before, we subtract the epact, we shall have the date of the Paschal full moon, which, though expressed as a day of March, will fall in April.

Easter is the Sunday following this full moon. It is desirable, therefore, to know the calendar letter corresponding to the day on which the full moon falls. When this day is the 21st March, this letter is always $C = 3$. The epact which brings the full moon to March 21st is, as we have seen, $= 23$. If the full moon falls on the 22d, the letter becomes $D = 4$, and the epact recedes from 23 to 22; or, generally, as the value of the calendar letter belonging to the Paschal moon increases, in the same manner the value of the epact diminishes; so that the sum of these two values is a constant quantity. This constant is, accordingly, $= 23 + 3 = 26$. And the rule for finding the calendar letter corresponding to the Paschal full moon is the following:

Subtract the epact from 26, and the remainder (after suppressing *sevens*) is the value of the calendar letter sought. But, inasmuch as an epact greater than 23 requires, for finding the day of Paschal moon, a constant increased from 44 to 74, so here, for the higher epacts, the constant 26 must be raised to 56, by the same addition (of 30) as in the former case. Accordingly, when the epact exceeds 23, subtract it from 56, and the remainder (with sevens suppressed) will give the calendar letter sought.

The Sunday following the Paschal full moon being Easter Sunday, we

may find *how many days later* it falls by subtracting the moon's calendar letter, found as above, from the Sunday Letter for the year; increasing this latter by seven, if necessary. The numerical difference thus found, added to the date of Paschal full moon, will give the day of the month upon which Easter falls.

The system of epacts thus devised by Clavius, considered as a means of fixing the time of Easter and the other movable feasts of the Church, cannot be commended for its simplicity or its adaptation to popular use, or even its fitness to reach the popular intelligence. It would seem, indeed, as if it had not been intended to be popularly understood; for certainly if the premeditated purpose of its author had been to devise a scheme for rendering a comparatively simple subject obscure, he could not have been more completely successful.

The consideration of epacts keeps before the mind constantly the idea of a retrogradation of dates. This is by no means so simple of conception as that of an advance. But the annual backward movement of a new moon through the space of eleven days is practically equivalent to a forward movement of nineteen days, since nineteen is the complement of eleven to thirty, and thirty is the number of days in each embolismic month (the last excepted, which, because it is the last, affects no computation *within* the cycle).

Again, the presentation continually before the mind, of the *new moon*, when we have nothing to do (ecclesiastically) with the new moon, except to take a day thirteen days later, is a source of quite unnecessary confusion. The direct and simple mode of finding the time of ecclesiastical *full* moon, which is what we *do* want to know—a mode, also, which has the merit of being easily intelligible—would have been, in the time of Clavius, and is now, to start from some *known date of full moon*, and to add nineteen days for each succeeding year (or eighteen for that last one in which the passage is made from the end of one cycle to the beginning of another), dropping out the thirties, as in the calculation of epacts. The result of this gives immediately the day itself required. The calculation is one which any person can make without difficulty; but if it were made in advance, for a whole cycle, and the numbers tabulated, these numbers would occupy no more space than the epacts do now, and they would be the things wanted, and not intermediate instrumentalities for finding these things. Let us take, for instance, as a starting point, the last year of the cycle in our present century. Its epact, we have seen, is 18. Subtracting this from 44, we have, as the date of Paschal full moon (which we may represent by P), the 26th of March. For the next following year there will be a full moon (as we are now passing from one cycle to the next) later by eighteen days. And for the years succeeding that, a full moon will follow *nineteen* days later each year. We shall then have the general expression,

$$\text{P} = 26 + 18 + 19\,(n - 1).*$$

$$\text{Or, P} = 25 + 19\,n = 10 + 15 + 15\,n + 4\,n.$$

$$\text{And therefore, P} = 10 + 15\,(n + 1) + 4\,n. \qquad \text{(A.)}$$

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*There is an analogy between this expression and one of the equations of Gauss, in his method for finding Easter, given by Delambre (*Astronomie Théorique et Pratique*, tom. i. p. 712), which is in the following form, viz.:

$$d = \Big((19a + M) \div 30\Big)_r,$$

in which a is the remainder left in dividing the given year of our Lord by 19, and is, therefore, evidently $= n - 1$. Though the formula is given without explanation, M is easily made out to be $= 44 + 1 - \epsilon'' + 18 - 22$. The subscript r indicates that the *remainder* of the division by 30 only is to be taken. It is consequently apparent that d is the number of days from March 22 to the *first Paschal day:* that is, to the first day on which, with this value of n, Easter can possibly fall; which day is, of course, the first day after Paschal full moon.

If l be put for the Calendar Letter of the first Paschal day, and Δ for the Dominical Letter of the year, we shall have, for the time of Easter referred to March,

$$\text{E} = 22 + d + \Delta - l;$$

to which *seven* must be added in case l exceeds Δ.

As the Calendar Letter of March 22 is $D = 4$. the value of l is of course $= d + 4$ (suppress-

Now when $n+1$ is even, $15(n+1)$ is a multiple of 30, and may be dropped. In this case, n itself is odd. Hence, when the number of the year in the cycle is odd,

$$P = 10 + 4n.$$

For example, the year 1872 is the 11th of the cycle, and for that year, $P = 10 + 44 = 54$; or, dropping 30, the Paschal full moon falls on the 24th of March.

It may be remarked that, if n is equal to or greater than 15, we may drop 15 from it before multiplying by 4; since $4 \times 15 = 60$, which is *twice thirty*. Thus, the year 1880 is the nineteenth of the cycle. Dropping 15 from 19, we have $P = 10 + 4 \times 4 = 26$, which is the same as stated above for $n = 19$.

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ing *sevens* if necessary). The peculiarity of Gauss's method consists in his ingenious general expression for the value of Δ, which, by analyzing his formulæ, we gather to be this:

$$\Delta = \delta + 4\left(\frac{Y}{7}\right)_r + 2\left(\frac{Y}{4}\right)_r;$$

where Y is the given year of our Lord, and δ is the Dominical Letter of some year less than Y, of which the numerical value is a multiple of 28; which is, therefore, at the same time a bissextile year. That this expression is universally true, may be shown as follows:

If N be taken to represent any number of years following the assumed bissextile year of which δ is the Dominical Letter, we shall have, for the Dominical Letter of the Nth year, the expression,

$$\Delta = \delta - N - \left(\frac{N}{4}\right)_q;$$

sevens being added in sufficient number to make the result positive. The subscript q denotes that the quotient of the division only is to be taken. This may be written otherwise as follows:

$$\Delta = \delta + 6N - \left(\frac{N}{4}\right)_q;$$

since the subtraction of a unit is equivalent, in its effect upon the Calendar Letter, to the addition of *six* units. The negative term here can have only the values $-1, -2, -3, -4$, etc.; and these, without affecting the value of Δ, may be replaced by $-8, -16, -24$, etc., since $-8 = -1 - 7$; $-16 = -2 - 2 \times 7$; $-24 = -3 - 3 \times 7$, etc.

Further, the positive term $6N$ may be written $= 4N + 2N$; and when $N = 4$, $N = 8$, etc., it will have the values $4N + 8$, $4N + 16$, etc., so that, in these cases, the equation may be written,

$$\Delta = \delta + 4N + 8 - 8;\ \Delta = \delta + 4N + 16 - 16, \text{ etc.,}$$

In other words, when $N = 4$, or any multiple of 4, the term $2N$ and the negative term neutralize each other. Hence, generally,

$$2N - \left(\frac{N}{4}\right)_q = 2\left(\frac{N}{4}\right)_r.$$

Moreover, when N is a multiple of 7, the term $4N$ may be suppressed without affecting Δ; so that, for the case in which N is a multiple of both 4 and 7, the equation simplifies itself to $\Delta = \delta$; and $4N$ is always equivalent to $4\left(\frac{N}{7}\right)_r$. Hence the general expression for Δ is this, viz.:

$$\Delta = \delta + 4\left(\frac{N}{7}\right)_r + 2\left(\frac{N}{4}\right)_r;$$

the *sevens* in the sum of these terms being suppressed.

Consequently,

$$\Delta - l = \delta + 4\left(\frac{N}{7}\right)_r + 2\left(\frac{N}{4}\right)_r - (d + 4).$$

Or,

$$\Delta - l = \delta + 4\left(\frac{N}{7}\right)_r + 2\left(\frac{N}{4}\right)_r + 6d + 24.$$

Now, if we put $Y = 28m + N$, m being integral, we shall have,

$$\left(\frac{N}{7}\right)_r = \left(\frac{Y}{7}\right)_r;\ \text{and}\ \left(\frac{N}{4}\right)_r = \left(\frac{Y}{4}\right)_r.$$

Accordingly, if $28m$ be the number denoting the bissextile year of which δ is the Dominical Letter, we may find the value of δ, for any assumed value of m, by the ordinary methods. This value will be the same for any other multiple of 28; as, for instance, for $28m'$; unless, between the values $28m$ and $28m'$, there intervene one or more non-bissextile centurial years

Suppose we take $m = 65$; then $28m = 1820$; and for 1820, $\delta = A = 1$. Then,

$$\Delta - l = 1 + 4\left(\frac{Y}{7}\right)_r + 2\left(\frac{Y}{4}\right)_r + 6d + 24;$$

Or, uniting the numerical terms and suppressing *sevens*,

$$\Delta - l = 4\left(\frac{Y}{7}\right)_r + 2\left(\frac{Y}{4}\right)_r + 6d + 4.$$

If $m = 68$, $28m = 1904$; and $\delta = B = 2$; whence the final term of the foregoing expression becomes 5 for years between 1899 and 2099, exclusive of the first of these years, and inclusive of the last. In Gauss's formulæ this variable numerical term is represented generally by the letter N, for which we have found a different provisional use above.

An improvement on Gauss's method was suggested in the year 1817, by Father Ciccolini,

But if $n + 1$ is odd (in which case n itself must be even) then the formula (A) foregoing may be written,

$$P = 10 + 15 + 15\,n + 4\,n.$$

Or (15 n being a multiple of 30),

$$P = 10 + 15 + 4\,n = 25 + 4\,n.$$

And here, as before, if n equals or exceeds 15, 15 may be dropped before multiplying.

These formulæ embrace the simple rules given in the first part of this paper for finding the time of Paschal full moon. If we had taken the epact of the last year of the cycle as it stood before 1700 (it was 19 instead of 18), we should have found $P = 25$ (instead of 26) for that year; and the result would have been to make the constants (10 and 25) in the final formulæ a unit less. Thus, for the 16th and 17th centuries,*

$$\text{When } n \text{ is odd, } P = 9 + 4n. \qquad \text{(B.)}$$

$$\text{And when } n \text{ is even, } P = 24 + 4n. \qquad \text{(B}'\text{.)}$$

During these centuries the secular correction in the General Table II of the Prayer Book was zero. The value of this correction, as it accrues, is to be added to the constants foregoing; and we see now why the number *nine*, subtracted from the lesser of these constants, will always (as stated in the foregoing pages) leave as a remainder the secular correction corresponding to the century found in the table of the Prayer Book just referred to.

Clavius laboriously reduced his whole system, including the necessary secular corrections for long periods, to tabular form. Delambre, in the first volume of his "History of Modern Astronomy," and De Morgan, in a paper contributed to the "Companion to the British Almanac" for 1845, have given rules for the calculation of all the elements which enter into the determination of Easter. The formulæ devised by the celebrated mathematician Gauss for the same purpose, with the modification proposed by Ciccolini, have been noticed in the note referred to on page ~~555~~.
29 Delambre also presented his rules in the form of algebraic expressions. The formula

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an ecclesiastic of Rome, the object of which was to render the term N a constant. It will be observed that the variability of N arises from the occurrence of non-bissextile centurial years. The most general form, therefore, of stating the value of Δ, or that which makes it true for all centuries, is the following:

$$\Delta = \delta - Y - \left(\frac{Y}{4}\right)_q + C - \left(\frac{C}{4}\right)_q,$$

in which C stands for the number of complete centuries, and Y, as before, stands for the entire number of years in the given year of our Lord. For the Julian Calendar, or old style, the value of Δ is always given by the first three terms of this expression. The final two, which correct for the omission of three leap years in four centuries, explain themselves. We have seen how to dispose of the terms containing Y. As for those dependent on C we add, in the first place, $7C$. These terms then become,

$$8C - \left(\frac{C}{4}\right)_q = 6\,C + 2C - \left(\frac{C}{4}\right)_q = 6\left(\frac{C}{7}\right)_r + 2\left(\frac{C}{4}\right)_r.$$

Hence, the value of Δ becomes,

$$\Delta = \delta + 4\left(\frac{Y}{7}\right)_r + 2\left(\frac{Y}{4}\right)_r + 6\left(\frac{C}{7}\right)_r + 2\left(\frac{C}{4}\right)_r.$$

If, now, δ be the Dominical Letter of the year zero, supposing the Gregorian computation to be extended so far back, the foregoing formula will give the Dominical Letter of any other year in any century, δ remaining constant. This constant value of δ is evidently that of every bissextile centurial year; and this we have already seen to be $= 1$.

Gauss uses the letter e to stand for the value of $\Delta - l$, with the *sevens* suppressed. Hence, for Easter, as referred to March, we have,

$$E = 22 + d + e.$$

From the manner in which the value of M, in the expression first cited above, is obtained, it is manifest that this value will be affected by the secular correction of the epact, increasing as the epact diminishes, and *vice versâ*. It will also be understood, after what has been said of the doubled epacts, XXIV and XXV, that when d is found $= 29$, it must always be taken $= 28$; and that when d is by formula $= 28$, it must be taken $= 27$ in case the Golden Number exceeds *eleven*.

* As stated in a former note, the constant term in finding P before the reformation of the calendar was *two*. Accordingly, for old style dates, we have the equations following:

$$\text{When } n \text{ is odd, } P = 2 + 4\,n.$$

$$\text{When } n \text{ is even, } P = 17 + 4\,n.$$

above given for the epact in the 16th century, viz., $\varepsilon = n + 10\,(n-1)$, was first stated by him.

Considering, then, that the epact, by the effect of the solar correction, is diminished three days in every 400 years, counted *after* 1600, Delambre proceeded to give the epact as affected by this correction thus (putting C for the number of complete centuries)

$$\varepsilon = n + 10\,(n-1) - \tfrac{3}{4}\,(\mathrm{C}-16) = n + 10\,(n-1) - (\mathrm{C}-16) + \tfrac{1}{4}\,(\mathrm{C}-16),$$

the *integral value* of the fractional expression only being used. Considering, also, that the lunar correction increases the epact one day every three hundred years, and that it was first applied in 1800, he finds that the effect of this correction, without regarding the irregularity which occurs at the end of every 25 years, may be expressed by the formula (putting c for correction),

$$c = \frac{\mathrm{C}-15}{3},$$

the *integral value* of the fraction only being used. Now, the eighth correction *after* 1800 (which centurial year marks the end of the period of supposed or imaginary corrections preceding the actual introduction of the reformation) would fall, according to this formula, on the centurial year 4200 (the twenty-fourth after 1800), whereas the theory requires that it should be deferred to the twenty-fifth; that is, to the centurial year 4300. In like manner, theory requires that the eighth correction after 4300 should be deferred to the twenty-fifth centurial year following that date, instead of falling upon the twenty-fourth.

Hence, it is necessary to place, in the numerator of the fraction expressing the lunar correction, as above, a negative fractional term which shall increase with the progress of time, and shall gain a unit in twenty-five years, becoming integral first in 4200. The effect of this term will be to reduce 42 to 41, and thus defer for a century the advance of the value of c. Such a term is

$$\frac{\mathrm{C}-17}{25},$$

Since $25 + 17 = 42$, and $\dfrac{42-17}{25} = 1$.

The complete expression for the value of c, therefore, is

$$c = \frac{\mathrm{C}-15 - \dfrac{\mathrm{C}-17}{25}}{3};$$

and as this goes to increase the epact, the general expression for the epact becomes

$$\varepsilon = n + 10\,(n-1) - (\mathrm{C}-16) + \frac{\mathrm{C}-16}{4} + \frac{\mathrm{C}-15 - \dfrac{\mathrm{C}-17}{25}}{3};$$

an expression sufficiently formidable.

But, though this degree of complication is necessary to the complete algebraic statement of all the conditions affecting the value of the quantity sought, yet, for practical purposes, the irregularity of long period may be disregarded, a special provision being made for it, to be separately applied. Then, if we put s to represent the combined effect of the secular corrections due to both sun and moon, we may state this effect as follows:

$$s = -\,(\mathrm{C}-16) + \frac{\mathrm{C}-16}{4} + \frac{\mathrm{C}-15}{3}.$$

Or, $s = -\,\mathrm{C} + 16 + \tfrac{1}{4}\,\mathrm{C} - 4 + \tfrac{1}{3}\,\mathrm{C} - 5 = -\,\mathrm{C} + \tfrac{1}{4}\,\mathrm{C} + \tfrac{1}{3}\,\mathrm{C} + 7$;

in which the three terms containing C must not be united, since the *integral values* of the fractional terms only are employed.

This is the secular correction of the *epact*. But, inasmuch as the date of full moon advances as the epact diminishes, and *vice versâ*, by the same

amount, this, with signs reversed, is also the secular correction of the constant in equation (B) which gives the value of P. To make that equation general, introduce S ($= -s$) to represent the correction, and we shall have,

$$P = 9 + S + 4n.$$

Or, substituting the value of $-s$ in place of S,

$$P = 9 + C - \tfrac{1}{4}C - \tfrac{1}{3}C - 7 + 4n = C - \tfrac{1}{4}C - \tfrac{1}{3}C + 2 + 4n.$$

And this shows us that the constant for the century is found, as the rule given earlier in this paper states, by subtracting from the centurial number its fourth part and its third part successively, and increasing the result by *two*. From what has just been said, we see also the reason for the special rules in relation to the centurial years 4200, 6700, 9200, etc., which follow the general rule referred to.

V. The fifth rule relates to finding the date of Easter Sunday, after that of the Paschal full moon has been ascertained. This will not require many words. It will be observed that the calendar letter, G, nearest preceding the earliest possible date of Easter (March 22), corresponds to the 18th day of March. If, then, to *eighteen* we add the numerical value of the Dominical Letter of the year, the sum will be the date of a Sunday. If this date exceeds that of Paschal full moon, it will be the date of Easter Sunday. If not, *seven* must be added often enough to obtain a date exceeding that of the Paschal full moon, each addition of seven giving, of course, a later Sunday. Thus the rule is justified.

These explanations have been protracted beyond the original intention. It has been found difficult, in less space, to present clearly the reasons on which the new rules are founded. That these rules are preferable, in respect to simplicity and facility of application, to any which have been heretofore proposed, will probably be admitted by any one who will take the trouble to compare them with those of Gauss and Ciccolini, or with those laid down by De Morgan in his article already referred to, which are probably the best hitherto published, or, finally, with the cumbrous formulæ of Delambre.

I have the honor to be, Reverend and dear Sir,

Your obedient servant,

The Rev. B. I. HAIGHT, S.T.D., LL.D. F. A. P. BARNARD.

V.

TABLE REFERRED TO IN RESOLUTION 3.

YEARS OF OUR LORD.	GOLDEN NUMBER.	THE EPACT.	SUNDAY LETTER.	EASTER-DAY.
1881............	1	0	B	April 17.
1882............	2	11	A	" 9.
1883............	3	22	G	March 25.
1884............	4	3	F E	April 13.
1885............	5	14	D	" 5.
1886............	6	25	C	" 25.
1887............	7	6	B	" 10.
1888............	8	17	A G	" 1.
1889............	9	28	F	" 21.
1890............	10	9	E	" 6.
1891............	11	20	D	March 29.
1892............	12	1	C B	April 17.
1893............	13	12	A	" 2.
1894............	14	23	G	March 25.
1895............	15	4	F	April 14.
1896............	16	15	E D	" 5.
1897............	17	26	C	" 18.
1898............	18	7	B	" 10.
1899............	19	18	A	" 2.

ADDENDA.

I. Rules for the Sundays after Trinity.

1. *To determine on what day any given Sunday after Trinity will fall*

Write in their order, as below, the names of the several months, from May to November, inclusive. Immediately under these, severally, write the terms of a numerical series beginning with 1, and increasing by the successive differences 5, 4, 4, 5, 4, 4; differences easily remembered by their symmetry of arrangement. Under these, again, write the three *even* numbers less than *seven*, the three *odd* numbers less than *seven*, and the zero, thus: 2, 6, 4, 1, 5, 3, 0—where the terms of the two triplets of significant figures, though not in regular arithmetical progression, are symmetrically arranged. These last numbers are to be called the *indices* of those above them. The result will then be as follows:

Months,	May,	June,	July,	Aug.,	Sept.,	Oct.	Nov.
Sundays,	1	6	10	14	19	23	27
Indices,	2	6	4	1	5	3	0

This little table, when once formed, may be preserved for permanent use; but it is so simple that it may be reconstructed, without much trouble, for each occasion.

Let, now, E stand for the date of Easter (referred to March); S, for the number of the Sunday of which the date is required; T, for the term in the Sunday series which is next less than S; and i, for the index of this term. Find, then, a numerical term, a, by the following equation:

$$a = 7(S - T) + i.$$

Then, d, the required date of the given Sunday, *as referred to the month standing over the term* T *in the small table* above, will be found by this formula:

$$d = \mathrm{E} + a.$$

Thus, in 1871, when Easter is 9th April = 40th March, we have, for the *twenty-first* Sunday after Trinity, 19 as the Sunday term, with 5 as its index.

Then, $S - T = 21 - 19 = 2$; and $7 \times 2 + 5 = 19 = a$.

Whence $\mathrm{E} + a = 40 + 19 =$ 59th September = 29th October = date required.

To show the operation of the rule in extreme cases, take the year 1818, when Easter was March 22; and the year 1886, when Easter will be April 25 = March 56.

In 1818, for the 18th Sunday after Trinity, we have $T = 14$, and $i = 1$. Then,

$$S - T = 18 - 14 = 4; \text{ and } 7 \times 4 + 1 = 29 = a.$$

Whence $\mathrm{E} + a = 22 + 29 =$ 51st August = 20th September = date required.

In 1886, for the 5th Sunday after Trinity, $T = 1$ and $i = 2$.

$$S - T = 5 - 1 = 4; \text{ and } 7 \times 4 + 2 = 30 = a.$$

$\mathrm{E} + a = 56 + 30 =$ 86th May = 55th June = 25th July = required date.

In case $S = T$, the solution is very simple; since $d = \mathrm{E} + i$, immediately.

Thus, in 1886, the 10th Sunday after Trinity gives,

$\mathrm{E} + i = 56 + 4 =$ 60th July = 29th August = required date.

2. *Given the date of any Sunday after Trinity, to determine its number.*

In this case, if d is not greater than E (referred to March), it must be increased by adding to it the number of days in the month preceding (or, if necessary, in the *two* months preceding), when the following will be true, viz.:

$$\left(\frac{d-\mathrm{E}}{7}\right)_r = i.$$

The index, i, points out, of course, the Sunday term, T. Then, if we put N for the number sought, and take n, an additional term found from the following, viz.,

$$n = \left(\frac{d-\mathrm{E}}{7}\right)_q,$$

we shall have finally,

$$N = T + n.$$

Thus, in 1818, the 19th of July = 49th of June was a Sunday, and

$$\left(\frac{d-\mathrm{E}}{7}\right)_r = \left(\frac{49-22}{7}\right)_r = \left(\frac{27}{7}\right)_r = 6 = i\text{; and } T = 6.$$

Then $\left(\frac{49-22}{7}\right)_q = \left(\frac{27}{7}\right)_q = n = 3$; and $N = T + n = 6 + 3 = 9$th Sunday after Trinity.

In 1886 the 21st November will be a Sunday. As Easter, in that year, will be April 25 = March 56, we must add 61 days for the two months preceding November, making d = 82d September. Then,

$$\left(\frac{d-\mathrm{E}}{7}\right)_r = \left(\frac{82-56}{7}\right)_r = \left(\frac{26}{7}\right)_r = 5 = i\text{; and } T = 19.$$

Also, $\left(\frac{26}{7}\right)_q = n = 3$; and $N = T + n = 19 + 3 = 22$d Sunday after Trinity.

With a little practice, this method may be employed mentally. For this purpose, the relation of the months to the Sunday terms and their indices may be fixed in the mind, by remembering that the middle month is August, and that the corresponding Sunday term, 14, is one half the sum of the extremes, since $1 + 27 = 28$, and $28 = 2 \times 14$. The middle index is also *unity*.

II. General Rules for Placing Days of the Month in the Calendar of the Week, and the Contrary.

1. *To determine the day of the week on which any given day of any month will fall.*

The Sunday Letter for the year is presumed to be known, or to have been found by the methods already given. The year is divided into four quarters, as usual, beginning with January, April, July, and October.

The months from April to October form what is called the *summer half;* those from October to April, the *winter half.*

The days of the week are numbered from *one* to *seven.* During the summer half, Sunday is the first day of the week, and Saturday the seventh. During the winter half, *Monday* is the first day, and Sunday the seventh.

The difference between the value of the Sunday Letter and *seven* is called the *complement* of the Sunday Letter.

The number of days by which any month exceeds four weeks—twenty-eight days—is called its *excess.*

The number of days by which any month falls short of five weeks—thirty-five days—is called its *supplement.*

The *excess* will always be either 2 or 3; the *supplement* will always be either 5 or 4. The excess and the supplement of *February* will always be *zero;* the added day of leap year being compensated by the change of Sunday Letter in the succeeding months.

Put, now, $d =$ day of month; $\delta =$ day of week; $\kappa =$ Sunday Letter ($\gamma\rho\acute{\alpha}\mu\mu\alpha$ $\kappa\upsilon\rho\iota\alpha\kappa\acute{o}\nu$); $k =$ complement of this letter; $e =$ the excess of the month, or sum of the excesses of the months, preceding the given month in the same quarter; and $\varepsilon =$ complement of $e =$ supplement or sum of supplements of the same months. Then this equation will be true:

$$\delta = \left(\frac{d + e + k}{7}\right)_r;$$

it being understood that $\delta = 0$, and $\delta = 7$, equally denote the last, or seventh day of the week.

Take, for example, July 4, 1776. In this year $\kappa = F = 6$, and $k = 1$. July being the first month in the quarter, $e = 0$. Then,

$$\delta = \left(\frac{4 + 0 + 1}{7}\right)_r = \text{5th day of the week} = \text{Thursday.}$$

The inauguration of Gen. Washington as first President of the United States took place on March 4, 1789. For this year, $\kappa = D = 4$; $k = 3$; and $e = 3$. Hence,

$$\delta = \left(\frac{4 + 3 + 3}{7}\right)_r = \text{3d day of the week} = \text{Wednesday;}$$

the first day during the winter half being Monday.

For the months of January and February in leap years, care must be taken to use the Sunday Letter belonging to those months. The rule for Sunday Letter heretofore given, finds, in leap year, the letter for March and the subsequent months. The letter for the two months preceding is one place more advanced in alphabetic order, or one unit higher in value. Thus, in 1776, $\kappa = G = 7$ for January and February.

Again, the birth of Gen. Washington occurred on the 22d February, 1732, new style, in which year the Sunday Letter was $E = 5$ after February; and $F = 6$ in February and January. Hence, $k = 1$, and $e = 3$, so that

$$\delta = \left(\frac{22 + 3 + 1}{7}\right)_r = \text{5th day of the week} = \text{Friday.}$$

Washington's birth occurred, however, before the adoption of the Gregorian reckoning; and in old style it is said to have taken place on the 11th February, 1731; the beginning of the year having then been fixed at March 25. In the application of these formulæ to dates in old style, it is necessary to add a unit to the number of the year for dates from January 1 to March 24, inclusive, before finding the Sunday Letter. Afterward, the process is the same as for new style.

Thus, stating Washington's birthday as February 11, 1732 (old style corrected for this purpose), we shall have $\kappa = A = 1$, for the months following February, and $\kappa = B = 2$ for February and January; whence $k = 5$, and

$$\delta = \left(\frac{11 + 3 + 5}{7}\right)_r = \text{5th day} = \text{Friday, as before.}$$

This formula may be expressed in words as follows:

To the month-date add the EXCESSES *of the months preceding (if any) in the same quarter, and the* COMPLEMENT OF THE SUNDAY LETTER. *The excess of sevens in the sum is the value of the day of the week.*

2. *To determine the days in any month which will fall on a given day of the week.*

For this case we have a formula which will easily be borne in mind from its analogy to the foregoing, viz.:

$$d = \left(\frac{\delta + \varepsilon + \kappa}{7}\right)_r.$$

The date thus determined will be the earliest in the month which can fall on the given day of the week. By adding sevens to this, others may be obtained—three always, and sometimes four.

As an example of the use of this formula, take the following: The next meeting of the General Convention of the Church in the United States will

take place on the first Wednesday in October, 1874. On what day of the month will this meeting occur?

In 1874, $\kappa = D = 4$; and in October, since this is a month of the winter half, Wednesday is the third day of the week. Also, since October is the first month of the quarter, $\varepsilon = 0$. Then,

$$d = \left(\frac{\delta + \varepsilon + \kappa}{7}\right)_r = \left(\frac{3 + 0 + 4}{7}\right)_r = 0;$$

signifying that Wednesday is the last day of September; so that the first Wednesday in October is October 7th.

The meeting of Congress takes place on the first Monday in December, annually. In 1872 this day will be the second of the month; for, since $\kappa = 6$; $\varepsilon = 9$; and Monday is the 1st day of the week,

$$d = \left(\frac{1 + 9 + 6}{7}\right)_r = \text{2d day of the month.}$$

The annual Commencement at Columbia College is held on the last Wednesday in June. Wednesday, during the summer half, is the fourth day of the week. In 1872, $\kappa = 6$; and for June, $\varepsilon = 9$; whence,

$$d = \left(\frac{4 + 9 + 6}{7}\right)_r = \text{5th day of June for the 1st Wednesday;}$$

and $5 + 7 + 7 + 7 = 26$th day of June for the last Wednesday = date required.

This formula may thus be reduced to words:

To the week-date add the SUPPLEMENTS *of the months preceding (if any) in the same quarter, and the* VALUE OF THE SUNDAY LETTER. *The excess of sevens in the sum is the* EARLIEST *month-date falling on the given day of the week.*

The following formulæ may be used without first finding the Sunday Letter:

Put q = *quaternial* = number of leap years (or of *fours*) in the incomplete century; r = *residual* = excess of fours in the same; $\rho = 7 - r$ = *complement of residual; r' = century-residual* = excess of fours in the number of complete centuries. Then, for day of week,

$$\delta = \left(\frac{d + e + r + 5(q + r') - 1}{7}\right)_r;$$

and, for day of month,

$$d = \left(\frac{\delta + \varepsilon + \rho + 2(q + r') + 1}{7}\right)_r.$$

In *leap years,* for dates in January and February (only), the numerical term in the foregoing formulæ should be 2 instead of 1; for the remaining months, from March to December inclusive, the term 1 is correct as it stands.

The foregoing are true for the Gregorian reckoning. The following are applicable to the Julian:

Put v = excess (ὑπεροχή) for the century; *i. e.*, the number of complete centuries, with *sevens* suppressed; $u = 7 - v$ = complement of excess; then,

$$\delta = \left(\frac{d + e + r + 5q + u - 3}{7}\right)_r; \text{ and}$$

$$d = \left(\frac{\delta + \varepsilon + \rho + 2q + v + 3}{7}\right)_r$$

In leap years, for dates in January and February (only), the numerical term in these formulæ should be 4 instead of 3.

In British (and British colonial) old style dates falling in January, February, or March (to March 24, inclusive), the number of the year should be increased by *unity* before applying the formulæ.

If, in the use of any of these formulæ, a negative value should be obtained, it must be made positive by adding *seven.*